NY-6 Systems of Linear and Quadratic Equations

Learning Standards for Mathematics

A.A.11 Solve a system of one linear and one quadratic equation in two variables, where only factoring is required.

A.G.9 Solve systems of linear and quadratic equations graphically.

Check Skills You'll Need

GO for Help Lessons 7-1, 7-2, and 10-4

1. Solve the system using substitution.
$x = y + 2$
$4x + y = 8$

2. Solve the system by graphing.
$y = 2x + 3$
$x = y$

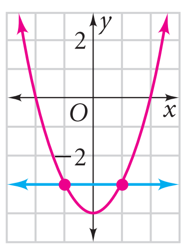
3. Solve $x^2 - 5x + 6 = 0$ by factoring.

1 Solving Systems Using Graphing

In Lesson 7-1, you solved systems of linear equations graphically and algebraically. A system of linear equations can have either one solution, no solutions, or infinitely many solutions. In Chapter 10, you solved quadratic equations graphically and algebraically.

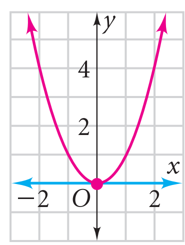
In this lesson, you will study systems of linear and quadratic equations. This type of system can have one solution, two solutions, or no solutions.

$y = x^2 - 4$
$y = -3$



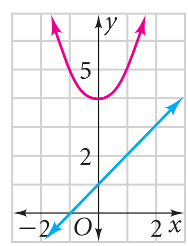
two solutions

$y = x^2$
$y = 0$



one solution

$y = x^2 + 4$
$y = x + 1$

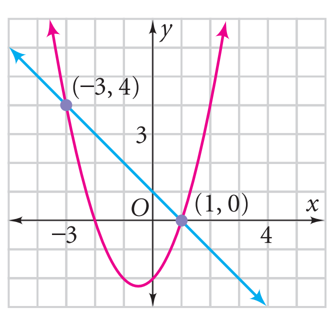


no solutions

1 EXAMPLE Solve by Graphing

Solve the following system by graphing. $y = x^2 + x - 2$
$y = -x + 1$

Graph both equations on the same coordinate plane. Identify the point(s) of intersection, if any.



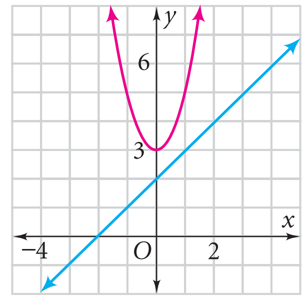
The points $(-3, 4)$ and $(1, 0)$ are the solutions of the system.

Quick Check

1 Solve the system by graphing. $y = 2x + 2$
$y = -x^2 - x + 2$

2 EXAMPLE Graphing to Count Solutions

Find the number of solutions for the system. $y = 2x^2 + 3$
$y = x + 2$



Step 1 Graph both equations on the same coordinate plane.

Step 2 Identify the point(s) of intersection, if any.

There are no points of intersection, so there is no solution to the system of equations.

Quick Check 2 Find the number of solutions for each system.

a. $y = x - 4$
$y = 2x^2 + x$

b. $y = x^2 - 6x + 10$
$y = 1$

2 Solving System Using Algebraic Methods

In Lesson 7-3, you solved linear systems using elimination. The same technique can be applied to systems of linear and quadratic equations.

3 EXAMPLE Using Elimination

Solve the following system of equations: $y = x^2 - 11x - 36$
$y = -12x + 36$

Step 1 Eliminate y.

$$\begin{array}{rl} y = & x^2 - 11x - 36 \\ -\ (y = & \quad\ - 12x + 36) \\ \hline 0 = & x^2 + x - 72 \end{array}$$

Subtract the two equations.

Subtraction Property of Equality

Step 2 Factor and solve for x

$0 = x^2 + x - 72$

$0 = (x + 9)(x - 8)$ **Factor.**

$x + 9 = 0$ or $x - 8 = 0$ **Zero-Product Property**

$x = -9$ or $x = 8$

Step 3 Find the corresponding y values. Use either equation.

$y = x^2 - 11x - 36$	$y = x^2 - 11x - 36$
$y = (-9)^2 - 11(-9) - 36$	$y = (8)^2 - 11(8) - 36$
$y = 81 + 99 - 36$	$y = 64 - 88 - 36$
$y = 144$	$y = -60$

The solutions are $(-9, 144)$ and $(8, -60)$.

Quick Check 3 Solve the system using elimination. $y = x^2 + 4x - 1$
$y = 3x + 1$

4 EXAMPLE Using Substitution

Solve the following system of equations: $y = x^2 - 6x + 9$ and $y + x = 5$.

Step 1 Solve $y + x = 5$ for y.

$$y + x - x = 5 - x \qquad \textbf{Subtract } x \textbf{ from both sides.}$$
$$y = 5 - x$$

Step 2 Write a single equation containing only one variable.

$$y = x^2 - 6x + 9$$
$$5 - x = x^2 - 6x + 9 \qquad \textbf{Substitute } 5 - x \textbf{ for } y.$$
$$5 - x - (5 - x) = x^2 - 6x + 9 - (5 - x) \qquad \textbf{Subtract } 5 - x \textbf{ from both sides.}$$
$$0 = x^2 - 5x + 4$$

Step 3 Factor and solve for x.

$$0 = (x - 4)(x - 1) \qquad \textbf{Factor.}$$
$$x - 4 = 0 \quad \text{or} \quad x - 1 = 0 \qquad \textbf{Zero-Product Property}$$
$$x = 4 \quad \text{or} \quad x = 1$$

Step 4 Find the corresponding y-values. Use either equation.

$$y = -x^2 + 4x + 1 \qquad\qquad y = -x^2 + 4x + 1$$
$$= -(4^2) + 4(4) + 1 \qquad\qquad = -(1^2) + 4(1) + 1$$
$$= 1 \qquad\qquad = 4$$

The solutions of the system are (4, 1) and (1, 4).

Quick Check 4 Solve the system using substitution. $y - 30 = 12x$

$y = x^2 + 11x - 12$

In Lesson 10-7, you used the discriminant to find the number of solutions of a quadratic equation. With systems of linear and quadratic equations you can also use the discriminant once you eliminate a variable.

5 EXAMPLE Using the Discriminant to Count Solutions

At how many points do the graphs of $y = 2$ and $y = x^2 + 4x + 7$ intersect?

Step 1 Eliminate y from the system. Write the resulting equation in standard form.

$$y = x^2 + 4x + 7$$
$$-\underline{(y = \qquad\qquad 2)} \qquad \textbf{Subtract the two equations.}$$
$$0 = x^2 + 4x + 5 \qquad \textbf{Subtraction Property of Equality}$$

Step 2 Determine whether the discriminant, $b^2 - 4ac$, is positive, 0, or negative.

$$b^2 - 4ac = 4^2 - 4(1)(5) \qquad \textbf{Evaluate the discriminant.}$$
$$= 16 - 20 \qquad \textbf{Use } a = 1,\ b = 4, \textbf{ and } c = 5.$$
$$= -4$$

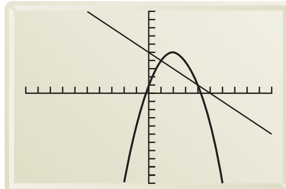
Since the discriminant is -4, there are no solutions. The graphs do not intersect.

Quick Check 5 At how many points do the graphs of $y = x^2 - 2$ and $y = x + 5$ intersect?

6 EXAMPLE Solve Using a Graphing Calculator

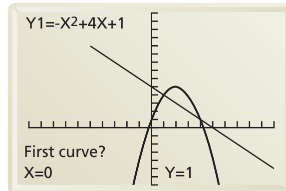
Solve the system of equations $y = -x^2 + 4x + 1$ and $y = -x + 5$ using a graphing calculator.

Step 1



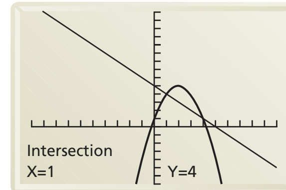
Enter $y = -x^2 + 4x + 1$ and $y = -x + 5$ into Y1 and Y2. Press GRAPH to display the system.

Step 2



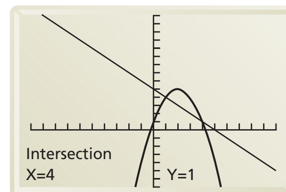
Use the CALC feature. Select 5: Intersect.

Step 3



Move the cursor close to a point of intersection. Press ENTER three times to find the point of intersection.

Step 4 Repeat Steps 2 and 3 to find the second intersection point.



The solutions of the system are (1, 4) and (4, 1).

Quick Check

6 Solve the system using a graphing calculator. $y = x^2 - 2$
$y = -x$

EXERCISES

For more exercises, see *Extra Skill and Word Problem Practice.*

Practice and Problem Solving

A Practice by Example

Examples 1 and 2 (pages NY 752 and NY 753)

GO for Help

Solve each system by graphing. Find the number of solutions for each system.

1. $y = x^2 + 1$
$y = x + 1$

2. $y = x^2 + 4$
$y = 4x$

3. $y = x^2 - 5x - 4$
$y = -2x$

4. $y = x^2 + 2x + 4$
$y = x + 1$

5. $y = x^2 + 2x + 5$
$y = -2x + 1$

6. $y = 3x + 4$
$y = -x^2$

Example 3 (page NY 753)

Solve each system using elimination.

7. $y = -x + 3$
$y = x^2 + 1$

8. $y = x^2$
$y = x + 2$

9. $y = -x - 7$
$y = x^2 - 4x - 5$

10. $y = x^2 + 11$
$y = -12x$

11. $y = 5x - 20$
$y = x^2 - 5x + 5$

12. $y = x^2 - x - 90$
$y = x + 30$

Example 4 (page NY 754)

Solve each system using substitution.

13. $y = x^2 - 2x - 6$
$y = 4x + 10$

14. $y = 3x - 20$
$y = -x^2 + 34$

15. $y = x^2 + 7x + 100$
$y + 10x = 30$

16. $-x^2 - x + 19 = y$
$x = y + 80$

17. $3x - y = -2$
$2x^2 = y$

18. $y = 3x^2 + 21x - 5$
$-10x + y = -1$

Example 5
(page NY 754)

Use the discriminant to find the number of solutions for each system.

19. $y = x^2 - 5x - 8$
$y = x$

20. $y = -x^2 - 3$
$y = 9 + 2x$

21. $y = -3x - 6$
$y = 2x^2 - 7x$

22. $y = 25x^2 - 9x + 2$
$y + 2 = 11x$

23. $y = -x^2 - 4x + 9$
$y + 5x = -7$

24. $4x^2 + 20x + 29 = y$
$8x + y + 20 = 0$

Example 6
(page NY 755)

Solve each system using a graphing calculator.

25. $y = x^2 - 2x - 2$
$y = -2x + 2$

26. $y = -x^2 + 2$
$y = 4 - 0.5x$

27. $y = x - 5$
$y = x^2 - 6x + 5$

28. $y = -0.5x^2 - 2x + 1$
$y + 3 = -x$

29. $y = 2x^2 - 24x + 76$
$y + 7 = 11$

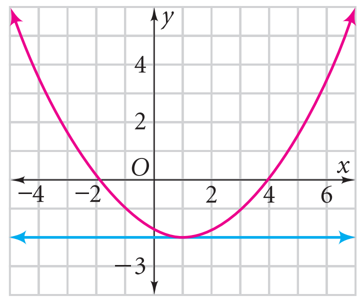
30. $-x^2 - 8x - 15 = y$
$-x + y = 3$

B Apply Your Skills

31. Critical Thinking The graph at the right shows a quadratic function and the linear function $y = d$.

a. If the linear function were changed to $y = d + 3$, how many solutions would the system have?

b. If the linear function were changed to $y = d - 5$, how many solutions would the system have?



Solve each system using either elimination or substitution.

32. $y = 2x^2 + 13x$
$y = -9 - 6x$

33. $y = -8x$
$y = 1 + 16x^2$

34. $y = x^2 + 9x - 91$
$x = \frac{y}{3}$

35. $y + 20x = 39$
$15 + 4x^2 + 9x = y$

36. $y = x^2 - 12x - 20$
$y = 25(4 - x)$

37. $5x^2 + 14x + 1 = y$
$-12 + y + 40x = 0$

38. Graphing Calculator The screen at the right shows the y- and x-values for the system $y = x^2 - 6x + 8$ and $y = x - 1$. Use the table to find the solutions of the system.

X	Y1	Y2
-1	15	-3
0	8	-2
1	3	-1
2	0	0
3	-1	1
4	0	2
5	3	3

X= -1

39. Writing Explain why a system of linear and quadratic equations cannot have an infinite number of solutions.

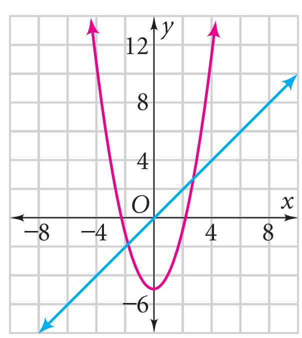
Use substitution and the quadratic formula to find the solutions of each system. Round your answers to the nearest hundredth.

40. $y = 2x^2 + 4x - 1$
$-5x + y = 5$

41. $2y + 4 = x$
$y + x^2 = 4$

42. $3x = y - 7$
$y = 6x^2 - 4x + 1$

43. The graph at the right shows the system $y = x^2 - 5$ and $y = x$. Find the values of x such that the y-values on the parabola are 10 units greater than the corresponding y-values on the line. Round your answers to the nearest hundredths.

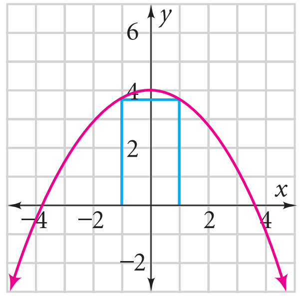


44. Critical Thinking Solve the system $y = x^2 + x + 25$ and $y = x$ using substitution. How can you tell that the system has no solutions *without* using graphing, the discriminant, or the quadratic formula?

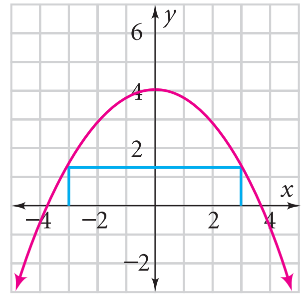
Challenge

45. **Geometry** The figures below show rectangles that are centered on the y-axis with bases on the x-axis and upper vertices defined by the function $y = -0.3x^2 + 4$. Find the area of each rectangle. Round to the nearest hundredth.

a.



b.



c. Find the x- and y-coordinates of the vertices of the square constructed in the same manner.

d. Find the area of the square. Round to the nearest hundredth.

NY REGENTS

Test Prep

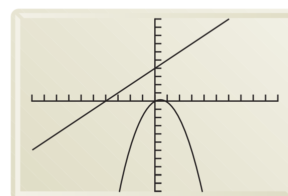
Multiple Choice

46. Which coordinate pair is a solution to the following system?

$y = x^2 + 2x - 2$
$y = x + 10$

A. (4, 14) **B.** (3, 13) **C.** (2, 12) **D.** (−3, 7)

47. The graph at the right shows the system $y = x + 4$ and $y = -x^2 + x$. How many solutions does the system have?



F. one solution
G. two solutions
H. no solutions
J. cannot be determined

Short Response

48. Use the discriminant to determine the number of solutions of the system.

$y = 49x^2 - 2x + 34$
$y + 30 = 100x$

49. Solve the system using substitution and factoring. Show your work.

$x^2 + 3x - 23 = y$
$\frac{y}{5} - 5 = x$

Mixed Review

Lesson NY-5

Tell whether each correlation is a causal relationship. Justify your answer.

50. Hours of computer use and television viewing have a negative correlation. Is this a causation?

51. The number of ice cream trucks in a town on a given day and the high temperature have a positive correlation. Is this a causation?

Lesson NY-4

Find each union or intersection. Let $A = \{1, 3, 6\}$, $B = \{2, 6, 8\}$, $C = \{x \mid x$ is an even number less than 6$\}$, and $D = \{x \mid x$ is a multiple of 3 less than 12$\}$.

52. $A \cup B$ **53.** $A \cap B$ **54.** $A \cap D$ **55.** $C \cap D$

56. $B \cap C$ **57.** $D \cup C$ **58.** $D \cup A$ **59.** $A \cap C$